During the last two decades of the 20th century, important changes have emerged in mathematics education. A major shift certainly is that mathematics is no longer mainly conceived as a collection of abstract concepts and procedural skills to be mastered, but primarily as a set of human sense-making and problem-solving activities based on mathematical modeling of reality. In accordance with this view, there is now rather general agreement that the ultimate goal of student learning is the acquisition of a mathematical disposition rather than of a set of isolated concepts and skills. These shifts in the perception of mathematics as a domain and of the objectives of mathematics education are attended with a fundamental change in the conception of learning mathematics, namely, from passive and decontextualized absorption of mathematical knowledge and skills acquired and institutionalized by past generations toward active construction in a community of learners of meaning and understanding based on the modeling of reality.

This view of mathematics as an active and constructive process, which is reflected worldwide in reform documents that put forward new standards for mathematics education (e.g., Cockcroft, 1982; National Council of Teachers of Mathematics, 1989; Treffers, De Moor, & Feys, 1989), implies that learners assume control and agency over their own learning and problem-solving activities. This means that self-regulation constitutes a
feature of effective learning and problem solving. This self-regulated conception of knowledge and skill acquisition contrasts sharply with many current educational practices in the modal mathematics classroom, where it is—mostly implicitly—assumed that regulatory activities are the responsibility of the teacher. In other words, in today's classrooms, external regulation of mathematics learning and problem solving by the teacher is the more typical situation and it prevails over self-regulation by the students. In addition, because children and youngsters apparently do not become self-regulated learners and problem solvers automatically and spontaneously, self-regulation of the processes of knowledge and skill acquisition and of problem solving is not only a major characteristic of productive learning, but it constitutes, at the same time and in itself, a main goal of a long-term learning process that, therefore, should be induced from an early age on.

From a conceptual point of view there is lack of clarity, and even confusion with respect to the concepts of metacognition and self-regulation (see also Pintrich, Wolters, & Baxter, in press; Zimmerman, 1994). In our opinion, the self-regulation of learning and problem solving is a form of action control that is characterized by the integrated regulation of cognition, motivation, and emotion (Boekaerts, 1997; Pintrich et al., in press; Snow, Corno, & Jackson, 1996). We endorse the viewpoint that self-regulation is a more comprehensive notion that, in addition to metacognitive processes, also encompasses motivational and emotional as well as behavioral monitoring and control processes (see e.g., Boekaerts, 1997; Zimmerman, 1995). We first elaborate our position by situating self-regulation in a theoretical framework of learning mathematics from instruction involving four essential components: acquiring a mathematical disposition as the ultimate goal, constructive learning processes as the road to the goal, powerful teaching–learning environments as support, and assessment as a basis for control and feedback. However, a review of the research literature shows that, until now, all the different aspects related to self-regulation of mathematics learning have not been investigated to the same degree. In the subsequent sections of this chapter, we focus on weaknesses observed in students with respect to metacognitive skills, metavolitional skills and beliefs as central aspects of a mathematical disposition, on the one hand, and attempts to develop and improve students self-regulation through systematic instructional interventions, on the other.

II. LEARNING MATHEMATICS FROM INSTRUCTION:
OUTLINE OF A THEORETICAL FRAMEWORK

The extensive amount of research on mathematics learning and teaching carried out over the past two decades (see Bishop, Clements, Keitel,
Kilpatrick, & Laborde, 1996; De Corte, Greer, & Verschaffel, 1996; Grouws, 1992; Nesher & Kilpatrick, 1990; Nunes & Bryant, 1997; Steffe, Nesher, Cobb, Goldin, & Greer, 1996) has resulted in an empirically underpinned knowledge base that provides us with the building blocks for a theory of learning mathematics from instruction, that can guide the analysis of the quality and the effectiveness of educational practices, but also the design of new and more powerful teaching–learning environments for the acquisition of worthwhile pedagogical objectives. More specifically, the outcomes of this research allow us to give better answers than before to the following questions that represent four components of a theory of learning from instruction (for more detail, see De Corte, 1995a; De Corte et al., 1996):

1. What has to be learned (theory of expertise)?
2. Which kind of learning/developmental processes are necessary to attain the intended goals (theory of acquisition)?
3. What are appropriate instructional methods and environments to elicit and continue these acquisition processes in students (theory of intervention)?
4. Which types of assessment instruments are required to evaluate the degree of attainment of the intended goals (a theory of assessment)?

The general answer to the first question that has emerged from the analysis of expertise in mathematics (see, e.g., De Corte et al., 1996; Schoenfeld, 1992) is that students should acquire a mathematical disposition. Such a disposition requires the mastery of five categories of aptitude:

1. A well-organized and flexibly accessible domain-specific knowledge base involving the facts, symbols, algorithms, concepts, and rules that constitute the contents of mathematics as a subject-matter field.
2. Heuristics methods, that is, search strategies for problem solving that do not guarantee, but significantly increase the probability of finding the correct solution because they induce a systematic approach to the task.
3. Metaknowledge, which involves knowledge about one's cognitive functioning (metacognitive knowledge), on the one hand, and knowledge about one's motivation and emotions that can be used to deliberately improve volitional efficiency (metavolitional knowledge), on the other.
4. Self-regulatory skills, which embrace skills relating to the self-regulation of one's cognitive processes (metacognitive skills or cognitive self-regulation), on the one hand, and of one's volitional processes (metavolitional skills or volitional self-regulation), on the other.
5. Beliefs about the self in relation to mathematical learning and problem solving, about the social context in which mathematical
activities take place, and about mathematics and mathematical learning and problem solving.

We know from other research (see, e.g., Cognition and Technology Group at Vanderbilt, 1997) that students often possess certain knowledge and skills they cannot access or use when necessary to solve a given problem. Acquiring a mathematical disposition should help to overcome this well-know phenomenon of inert knowledge. Therefore, the integrated mastery of the different kinds of knowledge (i.e., domain specific, metacognitive, metavolitional), skills, and beliefs should result in the development of a sensitivity for occasions when it is appropriate to use them and an inclination to do so. According to Perkins (1995), this sensitivity for situations and contexts, and the inclination to follow through both are fundamentally determined by the beliefs a person holds. A person's beliefs about what counts as a mathematical context and what he or she finds interesting or important have a strong influence on the situations he or she is sensitive to and whether or not he or she engages in them. The relevance of beliefs as a component of a mathematical disposition and their impact on mathematics learning is also echoed in the *Curriculum and Evaluation Standards for School Mathematics* of the (U.S.) National Council of Teachers of Mathematics (1989): “These beliefs exert a powerful influence on students' evaluation of their own ability, on their willingness to engage in mathematical tasks, and on their ultimate mathematical disposition” (p. 233). If acquiring a mathematical disposition functions as the ultimate goal of mathematics education, the development of appropriate beliefs in students seems to be as important as mastering different kinds of knowledge and self-regulatory skills.

As far as the second question is concerned, research has led to the identification of a series of characteristics of effective (mathematics) learning processes that should be explicitly taken into account when addressing the third question about interventions. These features can be summarized in the following definition (De Corte, 1995b): Learning is a constructive, cumulative, self-regulated, goal-oriented, situated, collaborative, and individually different process of knowledge building and meaning construction. Thus, as already alluded to in the Introduction, the constructivist perspective on learning involves pupils becoming self-regulated learners and problem solvers; that is, “metacognitively, motivationally, and behaviorally active participants in their own learning processes” (Zimmerman, 1989b, p. 4; see also Boekaerts, 1997). Self-regulated learners in school are able to manage and monitor their own processes of knowledge and skill acquisition; that is, they master and apply self-regulatory learning and problem-solving strategies on the basis of self-efficacy perceptions in view of attaining valued academic goals (Zimmerman, 1989a). Skilled self-regulation enables learners to orient themselves toward new learning
tasks and to engage in the pursuit of adequate learning goals; it facilitates the appropriate decision making during learning and problem solving, as well as the monitoring of an ongoing learning and problem-solving process by providing one’s own feedback and performance evaluations, and by keeping oneself concentrated and motivated. It also already has been established in a variety of content domains, including mathematics, that the degree of students’ self-regulation correlates strongly with academic achievement (Zimmerman & Risemberg, 1997).

As previously mentioned, the features of effective learning processes should orient inquiry that attempts to contribute to answering the third question by developing and testing a series of guiding principles for the design of powerful mathematics learning environments. In this respect, major guidelines that have emerged from the literature and are in line with the dispositional view of and the constructivist approach to mathematics learning are the following (for a more detailed discussion see De Corte et al., 1996):

1. Induce and support constructive, cumulative, and goal-oriented acquisition processes in students.
2. Enhance students’ self-regulation of their own learning processes.
3. Embed learning as much as possible in authentic contexts that are rich in resources and offer ample opportunities for interaction and collaboration.
4. Allow for the flexible adaptation of instructional and emotional support, taking into account individual differences among students.
5. Facilitate the acquisition of general learning strategies and problem-solving skills embedded within the mathematics curriculum.

Finally, a theory of assessment offers methods and techniques for the construction and application of proper assessment instruments that are compatible with the new view about the objectives and the nature of mathematics learning and teaching. In this respect, strong criticisms of traditional techniques and practices of educational testing, predominantly based on the multiple-choice item format, have led to the development of alternative forms of assessments that reflect more complex, real-life or so-called authentic performances (see, e.g., Lesh & Lamon, 1992; Lester, Lambdin, & Preston, 1997; Romberg, 1995; for a critical discussion of the notion of “authentic” assessment, see Terwilliger, 1997). At the same time the need to integrate assessment with teaching and the importance of assessment instruments to yield information to guide further learning and instruction have been emphasized (see, e.g., Glaser & Silver, 1994). From the perspective of self-regulating learning, it is important in this respect to stimulate in students the development of attitudes toward and skills in assessing their own mathematical learning processes and performances.
It becomes obvious from the preceding outline of a theory of learning mathematics from instruction that the notion of self-regulation occupies a central position throughout this theory. Indeed, self-regulatory skills are a major component of the mathematical disposition as the ultimate objective of mathematics education, but self-regulation constitutes as well a key characteristic of productive mathematics learning activities and processes. The adequate use of cognitive and volitional self-regulatory skills in a specific task and instructional context depends on the subjective perception and interpretation of that dynamically changing context based on the knowledge, beliefs, skills, and strategies one possesses; that is, the different components of a mathematical disposition (see Greeno, Collins & Resnick, 1996). Specifically students' beliefs play an important role in this ongoing interpretation process, and as such have a major influence on students' problem-solving behavior. Schoenfeld (1985) pointed out that "beliefs establish the context within which resources, heuristics and control operate" (p. 45). Self-regulated mathematical learning and problem solving then can be defined basically in terms of the effective use of metacognitive and metavolitional skills, but to grasp the whole reality of self-regulated mathematical learning and problem solving, the use of these skills has to be situated against a more complex personal and contextual background (see Figure 1).

Taking this perspective on self-regulated mathematics learning into account, powerful teaching–learning environments should focus on the progressive growth and development of self-regulatory skills in students, and instruments for the assessment of those skills should be elaborated, but at the same time one should not lose sight of the powerful personal background components (i.e., beliefs, knowledge, ...) that influence the adequate use of those skills. The next section documents that research has revealed serious flaws in students with respect to the key aspects of self-regulation; that is, metacognitive and metavolitional skills. We further discuss research that shows the important influence of student beliefs on mathematical learning and problem solving. Finally, we review a series of exemplary design experiments aiming at improving self-regulation skills in students through systematic intervention.

III. STUDENTS' FLAWS IN SELF-REGULATORY SKILLS AND BELIEFS

The importance of the self-regulatory components of a mathematical disposition contrasts sharply with the research results that already were published in the 1980s that showed substantial weaknesses in many students in this respect.
A. FLAWS IN THE REGULATION OF COGNITIVE PROCESSES

The importance of self-regulation during mathematical problem solving has been very well documented in a study by Schoenfeld (1985; see also 1992). Schoenfeld videotaped high-school and college students working in pairs on unfamiliar geometry problems during 20-minute sessions, and contrasted their solution processes with those of experts in mathematics. The following task is an example of the problems used in this study: "Consider the set of all triangles whose perimeter is a fixed number P. Of these, which has the largest area? Justify your answer as best as you can" (Schoenfeld, 1985, p. 301).

The protocols of the solution processes were parsed into episodes that represented different activities: reading the problem, analyzing, exploring, planning, implementing, and verifying. Time-line graphs were used to represent the course of the solution processes visually. Figure 2 shows a representative time-line graph of an expert solving a difficult problem. This figure demonstrates clearly that regulation of cognitive activities constitutes an essential component of expert problem solving. Indeed, this expert
spent a substantial amount of time analyzing the problem in an attempt to understand what it was about and planning the solution process. Moreover, the expert continually reflected on the state of his or her problem-solving process; this is indicated in the graph by the inverted triangles that represent explicit comments during the solution process (e.g., “Hmm, I don’t know exactly where to start here,” followed by 2 minutes of analyzing the problem).

This expert approach is in sharp contrast to the typical problem-solving behavior that emerged from the more than 100 protocols of the student pairs. Indeed, in about 60% of the solution attempts, cognitive self-regulatory activities that are so typical of expert problem solving were totally lacking. The typical strategy in those cases is illustrated in Figure 3: reading the problem, quickly deciding to follow a certain approach, and sticking to it without considering any alternative, despite evidence that no progress was being made.

Other comparative studies of weak and skilled problem solvers of different ages—carried out not only in the United States (e.g., Lester & Garofalo, 1982; Silver, Branca, & Adams, 1980), but in distinct parts of the world—also support the crucial role of cognitive self-regulation in mathe-
Stages, time spent on each stage, sequencing, and management activity by Novices KW and AM in Protocol 9.2.

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Mathematics learning and problem solving. For example, Gurova (1985) analyzed the solution processes of 11-year-old Russian low and high performers on a series of difficult word problems. The high performers were much more aware of their problem-solving activities: they could explain their solution methods better, they could justify their solution strategies more appropriately, and they were more accurate in predicting which problems they had solved correctly. In his well-known studies, Krutetskii (1976) also observed differences between elementary and secondary school students of different ability levels with respect to metacognitive activities during word problem solving. Similar results were reported in the Netherlands. Nelissen (1987) found in elementary school children that good problem solvers were better in terms of self-monitoring and reflection than poor problem solvers. Overtoom (1991) registered analogous differences between gifted and average students at the primary and secondary school levels. De Corte and Somers (1982) observed a strong lack of planning and monitoring of problem solving in a group of Flemish sixth graders, leading to poor performance on a word problem test. In summary, there is abundant evidence to show that cognitive self-regulation constitutes a major aspect of skilled mathematical learning and problem solving.
B. FLAWS IN THE REGULATION OF VOLITIONAL PROCESSES

Nowadays educational researchers also acknowledge more and more the influence of motivational and emotional aptitudes next to (meta)cognitive aptitudes on mathematical learning and problem solving (see, e.g., McLeod, 1989, 1992). The self-regulation of these aspects of the learning and the problem-solving process asks for a competence to monitor and control one's volitional processes (Kuhl, 1994). This refers to the knowledge and the skills to create and support an intention until goal attainment. Students' knowledge about their motivational and emotional states and the way they influence the problem-solving process form the precondition to change the mode of one's functioning (Kuhl & Goschke, 1994). Monitoring and controlling motivational and emotional processes always implies to some extent an awareness of and knowing about these processes and their constituents. However, deliberately regulating one's volitional processes encompasses more than motivational and emotional control. Other aspects such as attention control, planning, and impulse control (i.e., environmental control, time control, inhibition of competing impulses) have to be considered also (Kuhl, 1994).

To our knowledge, until now, research has not very well documented the way in which the use of specific metavolitional skills influences the outcome of mathematical learning and problem solving. We know little about the differences between expert and novice mathematicians with respect to the self-regulation of volitional processes. However, there are some research results that point to the relevance of volitional self-regulation for mathematical learning and problem solving. A study by Seegers and Boekaerts (1993) indirectly showed how motivational control could influence the results of mathematical problem solving. They investigated the influence of more general motivational beliefs (goal orientation, attributional style, self-efficacy) on task-specific cognitions (subjective competence, task attraction, personal relevance) and emotional state, as well as the way in which these general and task-specific variables determine the willingness to invest effort (learning intention) and task performance. A group of 162 fifth and sixth graders (ages 11 and 12) was presented with a number of tests and questionnaires to measure, respectively, pupils' mathematical and reasoning ability and all the abovementioned variables. Specific questionnaires were developed to assess the three different general motivational beliefs for this age group. To measure the task-specific variables, the authors applied Boekaerts' (1987) (Quasi) On-Line Motivation Questionnaire. This questionnaire, which consists of two parts, was presented to the pupils simultaneously with a self-made mathematical problem-solving test. The first part was administered just after they glimpsed at the problems in the test, and it involved questions about task-specific cognitions, emotional state, and learning intention. The sec-
ond part which measures emotional state, invested effort, result assessment, and attribution, was administered just after finishing the problems.

The results showed that motivational beliefs had only an indirect influence on mathematics performance through the task-specific appraisals and, more specifically, through the perceived competence in the task (subjective competence). Motivational beliefs also indirectly determined the learning intention and emotional state through task-specific appraisals, but there was no relationship found between learning intention and emotional state, on the one hand, and performance, on the other hand. From these results, one can infer that a high willingness to invest effort and a positive emotional state yield only positive effects on performance when they are linked with high subjective competence; this illustrates the complex way in which motivational and emotional aptitudes influence problem solving. These results as well as those from other studies done by the same research group (e.g., Vermeer, 1997) point indirectly to the relevance of motivational control as the ability to generate positive appraisals in the orientation stage of mathematical problem solving (Boekaerts, 1994). Students who are able to generate positive scenarios when confronted with a problem feel more confident and are willing to invest more effort, and as a consequence are likely to obtain better learning results.

We have discussed motivational control as one important aspect of the self-regulation of volitional processes. The other four aspects (i.e., emotional control, attention control, planning, and impulse control) may be as important, but have not received much attention yet in research on mathematical learning and problem solving. An exception is the study by McLeod, Metzger, and Craviotto (1989) on the affective reactions of expert and novice problem solvers. In a small-scale study, they interviewed four professors (experts) and four undergraduate students (novices) about their affective reactions while solving a series of mathematical problems. They came to the conclusion that both groups experienced the same emotions when they were stuck on a problem, feeling at times frustrated, aggregated, and disappointed, but whereas the novices reacted with quitting or stubbornly going on in the same way, the experts focused on staying calm and flexible. Although this exploratory study does not allow general conclusions, it illustrates that people handle emotions in different ways (emotional control) during mathematical problem solving and that this has a definite impact on their problem-solving processes and outcomes.

In summary, these and other, more general studies (see also Zimmerman & Martinez-Pons, 1986) show that students' ability to regulate certain volitional aspects of their behavior has an important influence on the course of the problem-solving process as well as on the results. Students who possess the necessary knowledge and skills to adequately regulate their volitional processes get less distracted, know when to concentrate on what, and know how to react adequately to negative appraisals
or negative experiences during problem solving without falling into dysfunctional patterns of behavior (see also Boekaerts, 1992; Messick, 1987). However, much more research is needed to study the different aspects of volitional self-regulation in a more systematic and detailed way, and especially to clarify further the interactions between these volitional processes and the cognitive and metacognitive processes, and how they determine the outcome of mathematical learning and problem solving.

C. FLAWS IN STUDENTS' BELIEFS

Currently, there is a substantial amount of research that shows that a variety of beliefs that students hold are important determinants of their learning, thinking, and performance (see, e.g., Boekaerts, 1997; Pintrich, Marx, & Boyle, 1993). This is certainly also the case with respect to mathematics (see McLeod, 1992). Schoenfeld (1985) has done pioneering work in this regard:

Belief systems are one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks. One's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on. (p. 45)

In other words, beliefs have a very strong influence on one's approach to and effort investment in mathematical learning and problem tasks. In this respect, it is important to notice that such beliefs are not necessarily conscious, but often implicit. Schoenfeld's view on the self-regulatory impact of beliefs on mathematics learning is echoed in the Curriculum and Evaluation Standards for School Mathematics of the (U.S.) National Council of Teachers of Mathematics (1989) as cited previously.

In the available literature, researchers usually distinguish between three kinds of student beliefs: beliefs about the self in relation to mathematical learning and problem solving, beliefs about the social context (e.g., the mathematics class), and beliefs about mathematics and mathematical learning and problem solving (see, e.g., McLeod, 1992). The foregoing discussion of the study of Seegers & Boekaerts (1993) already revealed the important indirect influence self-beliefs (or motivational beliefs) have on mathematical problem solving. Several other researchers (e.g., Ames & Archer, 1988; Fennema, 1989; Kloosterman, 1988) also addressed the influence of self-beliefs on mathematical problem solving, often reporting gender differences in relation to differences in performance (see also Vermeer, 1997). For instance, Fennema (1989) found that "males have more confidence in their ability to do mathematics, report higher perceived usefulness, and attribute success and failure in mathematics in a way that has been hypothesized to have a more positive influence on achievement" (p. 211). Most of these studies (except Vermeer, 1997),
however, do not incorporate task-specific variables and as such do not allow us to differentiate between direct or indirect effects of these beliefs.

In addition to these self-beliefs, more and more research points out that students' problem-solving behavior in a specific context is governed by their beliefs about expectations, rules, and norms that are linked with that situation. The relevance of context beliefs is illustrated indirectly by the well-known ethnomathematic studies in which everyday mathematical problem-solving activities of children and adults were contrasted with their solution strategies in a formal school context. For instance, Nunes, Schliemann, and Carraher (1993) showed that Brazilian street vendors were very skillful, quick, and accurate at calculating how much one had to pay for a number of coconuts and that they used a variety of informal, invented strategies. However, confronted with isomorphic textbook word problems, the same children used the formal, algorithmic procedures learned in school and they committed many more errors.

Finally, students' beliefs about mathematics and mathematics learning and problem solving determine in a fundamental way their problem-solving behavior. It is argued that, probably as a consequence of current educational practices, students acquire beliefs relating to mathematics that are naive, incorrect, or both, but have mainly a negative or inhibitory effect on their learning activities and approaches to mathematics problems. For instance, according to Greeno (1991), most students learn from their experiences in the classroom that mathematics knowledge is not something that is constructed by the learner, either individually or in group, but is a fixed body of received knowledge. In a similar way, Lampert (1990) characterized the common view about mathematics as follows: Mathematics is associated with certainty, and with being able to give quickly the correct answer; doing mathematics corresponds to following rules prescribed by the teacher; knowing math means being able to recall and use the correct rule when asked by the teacher; and an answer to a mathematical question or problem becomes true when it is approved by the authority of the teacher. According to Lampert (1990), those beliefs are acquired through years of watching, listening, and practicing in the mathematics classroom.

Empirical support for these claims was, for instance, reported in an article by Schoenfeld (1988) with the strange title, "When good teaching leads to bad results: The disasters of ‘well-taught’ mathematics courses." Schoenfeld made a year-long intensive study of one 10th-grade geometry class comprised of 20 pupils, along with periodic data collections in 11 other classes (210 pupils), involving observations, interviews with teachers and students, and questionnaires relating to students' perceptions about the nature of mathematics. The students in those classrooms scored well on typical achievement measures, and the mathematics was taught in a way that generally would be considered good teaching. Nevertheless, it was found that students acquired debilitating beliefs about mathematics and
about themselves as mathematics learners, such as "all mathematics problems can be solved in just a few minutes" and "students are passive consumers of others' mathematics." It is obvious that such misbeliefs are not conducive to a mindful and persistent approach to new and challenging problems. Other strange beliefs that have been observed in pupils are that mathematics problems have one and only one right answer; formal mathematics has little or nothing to do with real thinking or problem solving; the mathematics learned in school has little or nothing to do with the real world (see, e.g., Schoenfeld, 1992).

In our own research, we also have observed that upper primary school children are affected by the belief that real-world knowledge is irrelevant when solving mathematical word problems. In the basic study (Verschaffel, De Corte, & Lasure, 1994), a paper-and-pencil test consisting of 10 pairs of problems was administered collectively to a group of 75 fifth graders (10- and 11-year-old boys and girls). Each pair of problems consisted of a standard problem (i.e., a problem that can be solved by straightforward application of one or more arithmetic operations with the given numbers; e.g., "Steve bought five planks of 2 meters each. How many 1-meter planks can he saw out of these planks?") and a parallel problem in which the mathematical modeling assumptions are problematic, at least if one seriously takes into account the realities of the context called up by the problem statement (e.g., "Steve bought four planks of 2.5 meters each. How many 1-meter planks can he saw out of these planks?"). An analysis of the pupils' reactions to the "problematic" tasks yielded an alarmingly small number of realistic responses or comments based on the activation of real-world knowledge (responding to the problem about the 2.5-meter planks with "8" instead of "10"). Indeed, only 17% of all the reactions to the 10 "problematic" problems could be considered as realistic, either because the realistic answer was given or the nonrealistic answer was accompanied by a realistic comment (e.g., with respect to the planks problem, some pupils gave the answer "10," but added that Steve would have to glue together the four remaining 0.5-meter pieces two by two).

This phenomenon of nonrealistic mathematical modeling and problem solving has been replicated in several other studies in which the same or a similar set of problematic items was administered to students from many different countries under largely the same testing conditions (for an overview, see Greer & Verschaffel, 1997). Interestingly, in some of these additional studies, interviews were carried out with some of the students, and they revealed that a significant number of students were able to articulate an awareness of a difference between conventional answers expected in the context of school mathematics and answers appropriate to real situations. One 10-year-old from a study by Greer (reported in Greer & Verschaffel, 1997) commented as follows in response to the interviewer's question as to why he did not make use of realistic considerations when solving the problematic items in the context of a school mathematics test:
"I know all these things, but I would never think to include them in a maths problem. Math isn’t about things like that. It’s about getting sums right and you don’t need to know outside things to get sums right.” Moreover, additional studies in our center (De Corte, Verschaffel, Lasure, Borghart, & Yoshida, in press), as well as by other European researchers (see Greer & Verschaffel, 1997), have shown that this misbelief about the role of real-world knowledge during word problem solving is very strong and resistant to change, and, moreover, that it is paralleled by a similar tendency among (future) teachers, as reflected by their own preferred spontaneous solutions to word problems and their way of evaluating realistic and unrealistic pupil answers. This finding supports the view that the opinions of the teachers themselves about doing and learning mathematics are at least partially responsible for the development in students of misbeliefs that have a negative impact on the regulation of their problem-solving approach and strategies.

In this section we have shown first that students’ metacognitive and metavolitional skills are important self-regulatory determinants of their learning, thinking, and problem solving. Second, we pointed to the fundamental way in which students’ beliefs influence their mathematical learning and problem solving. In this respect, it is striking that with respect to these aspects, the present situation in mathematics classrooms is rather bleak: many students, especially the weaker ones, lack the appropriate cognitive and volitional self-regulatory skills, and present teaching practices seem to induce in students mainly negative beliefs about mathematics as a domain and about themselves as learners of mathematics. It is useful to remark here that this unfavorable picture with respect to mathematics is not unique, but rather representative for school education today. For instance, Berry and Sahlberg (1996) studied the views about learning of 193 pupils (aged 15) in five schools in England and Finland. They took as a starting point De Corte’s (1995b) model of the features of good learning as a constructive, cumulative, self-regulated, goal-oriented, situated, and collaborative process of knowledge building and meaning construction. A major finding of this investigation was that “most pupils have ideas of learning that can be described by the transmission model and are quite difficult to fit with De Corte’s model …… This is to say that our pupils’ ideas of learning and schooling reflect the static and closed practices of the school” (Berry & Sahlberg, 1996, p. 33). The authors add that this view of the pupil as a passive and externally regulated absorber of information is in accordance with similar findings reported by other researchers for teachers and adult students.

This state-of-the-art perspective raises the important question of whether it is possible to design and implement powerful instructional environments in which students acquire productive self-regulating strategies and develop more positive beliefs relating to learning and teaching
IV. FOSTERING STUDENTS' SELF-REGULATION IN POWERFUL MATHEMATICS LEARNING ENVIRONMENTS

Notwithstanding the unfavorable picture of mathematics educational practice that emerges from the preceding section, it is the case that since the 1980s researchers have undertaken intervention studies aimed at improving students' self-regulation skills in mathematics through appropriate instruction. A number of these investigations have been conducted within the framework of a specific theoretical perspective on self-regulation and have a rather highly experimental character that focuses on the testing of hypotheses and predictions that derive from that theoretical approach. The most significant example of this kind of work is the series of studies by Schunk (1998) on the use of modeling for teaching primary school children to self-regulate the practice of mathematical skills, set up in the framework of the social–cognitive view of self-regulation. Other scholars have carried out more ecologically valid design experiments that are less closely related to a specific theoretical perspective on self-regulation, but aim at improving certain important aspects of cognitive self-regulation. As a whole, these research endeavors open perspectives for the implementation of a self-regulated conception of learning, and, as argued by Romberg (1992), the outcomes of that research call for a radical reform of mathematics learning and instruction, as well as a fundamental change in people's views and beliefs about mathematics and mathematics education. In this section we review four representative examples of the second category of studies, namely, design experiments focusing mostly on fostering students' cognitive self-regulation. These design experiment are an investigation by Schoenfeld (1985, 1992) at the college level, by Lester, Garofalo, and Kroll (1989) in seventh-grade classes, by the Cognition and Technology Group at Vanderbilt (1996, 1997) in the upper primary school, and by Verschaffel et al., (in press) with fifth graders.

A. TEACHING METACOGNITIVE AND HEURISTIC STRATEGIES IN GEOMETRY

Schoenfeld's (1985, 1992) work relating to geometry offers an excellent example of an attempt to create a powerful learning environment for teaching heuristic methods embedded in a cognitive self-regulation strategy for problem solving. The starting point of his intervention work was the observation reported earlier (see Section III.A) that metacognitive activities that constitute an essential characteristic of expert problem
solving are mostly lacking in students’ solution processes. Therefore, he designed a learning environment focused on the strategic aspects of problem solving. In this respect, Schoenfeld argues that it is not sufficient to teach students isolated heuristic procedures. Indeed, such heuristics are often inert, because students are unable to find and decide which heuristic is appropriate to solve the problem at hand. That is why it is necessary to teach heuristics in the context of a metacognitive strategy that supports the learner in selecting the right heuristic method to unravel and solve a given problem.

Schoenfeld (1985) elaborated such a regulatory strategy consisting of five stages:

1. Analysis oriented toward understanding the problem by constructing an adequate representation.
2. Designing a global solution plan.
3. Exploration oriented toward transforming the problem into a routine task.
4. Implementing the solution plan.
5. Verifying the solution.

According to Schoenfeld (1985), “the design stage is not really a separate phase, but something that pervades the entire solution process; its function is to ensure that you are engaged in activities most likely (as best as you can tell at that time) to be profitable. Most generally, it means keeping a global perspective on what you are doing and proceeding hierarchically” (p. 108). As such it can be viewed as the self-regulatory component par excellence of the strategy.

The exploration phase constitutes the heuristic heart of this strategy. Indeed, it is especially in transforming a problem into a routine task that heuristic methods are very helpful. However, such strategies are also useful in the problem analysis and verification stages. As an illustration, Table 1 lists a series of heuristic procedures that can be used in those three phases.

The instructional model that underlies Schoenfeld’s learning environment reflects major features of the so-called cognitive apprenticeship approach to learning and teaching (Collins, Brown, & Newman, 1989). From the beginning, Schoenfeld orients his students toward the five-stage regulatory strategy as a whole, albeit in a schematic form. Then the different stages are treated consecutively, and the corresponding relevant heuristics are explained and practiced. In this respect, modeling is extensively used to demonstrate how an expert selects and applies heuristic methods. Afterward, the students are given ample opportunities to practice those methods under the guidance of the teacher, who encourages them to use certain heuristics, gives hints, provides immediate feedback, and, if necessary, helps with the execution of some parts of the strategy that the students cannot carry out independently (scaffolding). These different
TABLE 1 The Most Important Heuristics in Schoenfeld's (1985) Problem-Solving Strategy

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Exploration</th>
<th>Verification</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Draw a diagram if at all possible</td>
<td>A. Consider essentially equivalent problems by replacing conditions by equivalent ones, by recombining the elements of the problem in different ways, by introducing auxiliary elements, or by reformulating the problem</td>
<td>A. Check whether all pertinent data were used and whether the solution conforms to reasonable estimates, predictions, or both</td>
</tr>
<tr>
<td>B. Examine special cases of the problem</td>
<td>B. Try to decompose the problem in subgoals, and work on them case by case</td>
<td>B. Check whether the solution could have been obtained differently</td>
</tr>
<tr>
<td>C. Try to simplify the problem</td>
<td>C. Look for a related or analogous problem where the solution method may be useful with a view to solving the new problem</td>
<td></td>
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</tbody>
</table>

forms of support and external regulation are gradually phased out as the students take over more and more agency for their problem-solving process; that is, become more and more self-regulated learners and problem solvers. In addition to modeling and whole-class teaching, Schoenfeld also uses small-group problem solving. Acting as a consultant, he regularly asks three questions during group activities:

1. What exactly are you doing? (Can you describe it precisely?)
2. Why are you doing it? (How does it fit into the solution?)
3. How does it help you? (What will you do with the outcome when you obtain it?)

Asking these questions serves two purposes: It encourages students to articulate their problem-solving strategies and it induces reflection on those activities; in other words, it stimulates and fosters cognitive regulation skills. The ultimate goal of this intervention is that students spontaneously and individually ask themselves the three questions, and in doing so regulate and monitor their own cognitive processes.

In a series of studies, Schoenfeld (1985, 1992) showed that college students can acquire the cognitive self-regulation strategy, as well as the embedded heuristics. Indeed, as a result of instruction, students' problem-solving approaches became more expertlike (see Figure 2). As reported previously, before instruction, 60% of the solution processes by students working in pairs lacked any sign of self-regulation; after the course, less than 20% of the solution attempts were of that type. The number of correct solutions increased accordingly.
Schoenfeld's approach clearly implies that self-regulatory skills and heuristics are taught. However, his learning environment is nevertheless essentially constructivist in nature. Indeed, the teacher does not impart problem solutions nor impose solution strategies, but supports students in their attempts at understanding problems, in reflecting on their methods and strategies, and in internalizing valuable self-regulation skills. In this respect, Schoenfeld's approach to the teaching of problem solving has developed in an interesting way: At present, the five stages of the control strategy delineated previously and the heuristics involved in it, are no longer taught in the same very structured and explicit way as was initially the case, but their importance and usefulness are highlighted when relevant and appropriate in the course of classroom discussions (Schoenfeld, 1987). In such an environment, student are not learning about mathematics, but they are doing mathematics. This involves learning how to use the tools of mathematics, and leads to the acquisition not only of mathematical concepts, but also of a mathematical view of the world and a sense of mathematical practice and culture (Schoenfeld, 1992). In summary, a real mathematical disposition, including cognitive self-regulation of mathematical learning and thinking, is encouraged and fostered.

B. TEACHING COGNITIVE SELF-REGULATORY SKILLS TO SEVENTH GRADERS

According to Lester et al. (1989), successful problem solving depends on five broad, interdependent categories of factors: knowledge, control, affects, beliefs, and contextual factors. In this intervention study they focus on metacognition that involves the knowledge and control one has of one's cognitive functioning. The control aspect refers to the regulation of one's cognitive processes and behavior during problem solving, and is more precisely defined as follows:

In particular, control has to do with the decisions and actions undertaken in analyzing and exploring problem conditions, planning courses of action, selecting and organizing strategies, monitoring actions and progress, checking outcomes and results, evaluating plans and strategies, revising and abandoning unproductive plans and strategies, and reflecting upon all decisions made and actions taken during the course of working on a problem. (Lester et al., 1989, p. 4)

To explore the effect of instruction on students' cognitive self-regulation of their problem-solving processes, Lester et al. (1989) designed a learning environment that involved (1) creating opportunities for students to practice in the use of a number of valuable heuristic strategies (strategy training), (2) helping students to become more aware of the strategies and procedures they use to solve problems (awareness training), and (3) training students to monitor and evaluate their actions during problem solving (self-regulation training).
More specifically, the learning environment consisted of (1) a set of appropriate problems and tasks, and (2) a series of lesson plans with teacher roles and activities. Similar to Schoenfeld's approach, the selection of problems and teaching activities was based on a theoretical framework of skilled problem solving, encompassing a series of heuristic strategies (i.e., guess and check, look for a pattern, work backward, draw a picture, make a table, simplify the problem) embedded in an overall cognitive regulation strategy composed of four stages, namely, orientation, organization, execution, and verification (see Table 2 for a specification of the four categories). These stages correspond largely to those that occur in Schoenfeld's model (1985): analysis, exploration, implementation, and verification.

Two broad types of problems were used in the study: routine and nonroutine problems. Routine problems were typical multistep word problems intended to provide pupils with practice in translating verbal prob-

### Table 2: A Cognitive-Metacognitive Framework for Studying Mathematical Performance

<table>
<thead>
<tr>
<th>Orientation: Strategic behavior to assess and understand a problem</th>
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</thead>
<tbody>
<tr>
<td>A. Comprehension strategies</td>
</tr>
<tr>
<td>B. Analysis of information and conditions</td>
</tr>
<tr>
<td>C. Assessment of familiarity with task</td>
</tr>
<tr>
<td>D. Initial and subsequent representation</td>
</tr>
<tr>
<td>E. Assessment of level of difficulty and chances of success</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Organization: Planning of behavior and choice of actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Identification of goals and subgoals</td>
</tr>
<tr>
<td>B. Global planning</td>
</tr>
<tr>
<td>C. Local planning (to implement global goals)</td>
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</table>

<table>
<thead>
<tr>
<th>Execution: Regulation of behavior to conform to plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Performance of local actions</td>
</tr>
<tr>
<td>B. Monitoring of progress of local and global plans</td>
</tr>
<tr>
<td>C. Trade-off decisions (e.g., speed vs. accuracy, degree of elegance)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Verification: Evaluation of decisions made and outcomes of executed plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Evaluation of orientation and organization</td>
</tr>
<tr>
<td>1. Adequacy of representation</td>
</tr>
<tr>
<td>2. Adequacy of organizational decisions</td>
</tr>
<tr>
<td>3. Consistency of local plans with global plans</td>
</tr>
<tr>
<td>4. Consistency of global plans with goals</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Adequacy of performance of execution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Adequacy of performance of actions</td>
</tr>
<tr>
<td>2. Consistency of actions with plans</td>
</tr>
<tr>
<td>3. Consistency of local results with plans and problem conditions</td>
</tr>
<tr>
<td>4. Consistency of final results with problem conditions</td>
</tr>
</tbody>
</table>

*aBased on the work of Lester et al. (1989).*
lems posed in real-world contexts into mathematical expressions. Three kinds of nonroutine tasks were administered: problems with superfluous information, problems with insufficient information, and process problems. To solve this latter category of tasks, the problem solver needs to do something more than just translate text in a mathematical expression, apply an algorithm, or perform computations. Illustrative of this category of nonroutine problems is the following: “A caravan is stranded in the desert with a 6-day walk back to civilization. Each person in the caravan can carry a 4-day supply of food and water. A single person cannot carry enough food and water and would die. How many people must start out so that one person can get help and the others can get back to the caravan safely?” The different types of tasks were included because each kind seemed especially suited to exemplify the need for and provide practice with particular aspects of the described cognitive/metacognitive framework.

Instruction focused on solving problems in small groups, in combination with whole-class discussions, and individual assignments. During these classroom activities, the teacher had to fulfill three different, but closely related roles: (1) serve as external monitor during problem solving, (2) encourage discussion of behavior considered important for the internalization of the heuristic and cognitive regulation skills, and (3) model good executive behavior. As a more specific guideline for the teacher, a set of 10 teaching actions before, during, and after the problem-solving activity was provided, together with their purposes. Examples of these actions are the following:

- Before: Use a whole-class discussion about understanding the problem.
- During: Observe and question students to determine where they are in the problem-solving process.
- After: Relate the problem to previously solved problems and discuss or have students solve extensions of the problem.

During the intervention, a chart with problem-solving tips was presented (see Table 3) for use by the teacher and the pupils. These tips reflect in a very specific manner the major aspects of the cognitive regulation strategy.

It is obvious that the instructional model underlying Lester et al.’s learning environment shares major characteristics with the approach of Schoenfeld, especially the use of modeling good problem-solving behavior and stimulation of small-group and whole-class discussion focusing on articulation and evaluation of problem-solving processes and activities.

The instructional program was realized by one of the investigators with a regular-level and an advanced-level seventh-grade class during about 15 hours spread over 12 weeks. Before, during, and after the instruction,
TABLE 3 Problem Solving Tips

Understanding the problem
A. Read the problem carefully; often you should read it two or more times.
B. Be sure you understand what the question is asking; ask yourself: "Do I understand what I am trying to find?"
C. If you are not sure you understand the problem, draw a picture or a diagram of the information.
D. Write down all the important information and the question; these are called what I know and what I want to find.

Solving the problem
A. Explore the problem to get a good "feel" for what the problem is about.
B. Do not do anything hard until you have tried easy ideas first; if easy things do not help, then you may need to do something more complicated.
C. When you do not have any idea of what to do, try to make a good guess and then check it out with the important data.
D. Use the strategies that you have learned; for example,
   - Draw a picture
   - Guess and check
   - Look for a pattern
   - Make a table
   - Work backwards
   - Simplify the problem

Getting an answer and evaluating it
A. Be sure to check your work along the way, not just at the end; you may be able to avoid some unnecessary work by finding a mistake early.
B. Be sure that you used all the important information.
C. Write your answer in a complete sentence; this makes it easier to decide if the answer is reasonable.
D. Ask yourself, "Does my answer make sense?"

*Lester et al. (1989) employed a large set of assessment instruments, including written tests, clinical interviews, observations of individual and pair problem-solving sessions, and videotapes of classroom instruction.

The results on the written pretest and posttest, involving a set of new routine and nonroutine word problems, revealed positive effects of the learning environment on pupils' problem-solving skills. Both the regular class and the advanced class realized a considerable overall gain in total score from pretest to posttest, albeit the progress was not as great as expected. Interestingly, large individual differences were observed: although the scores of most pupils in both classes increased from pretest to posttest, the scores of a significant number of pupils remained the same or even decreased.

The results of the individual interviews before and after instruction revealed that among the four categories of activities in the cognitive regulation strategy, orientation, which refers to different types of strategic behavior to assess and understand a problem (see Table 2), had the most important effect on students' problem solving. However, no substantial
differences were observed in this respect between pupils' regulatory activities before and after instruction. One explanation for this lack of effect of the learning environment is that the total time of the intervention was not long enough to produce significant changes. Moreover, problem-solving instruction was alternated with regular mathematics teaching, which reinforced pupils' "non-self-regulatory old habits" and may have been perceived as more important. Finally, the learning environment may not have been implemented with a high degree of fidelity. With regard to this latter explanation, several observations of the lessons revealed some weaknesses of the instruction and suggest, at the same time, possible improvements. First, the teachers experienced serious trouble maintaining their role of model, facilitator, and monitor in the face of classroom reality, especially when pupils had difficulties with basic aspects of the subject matter. Second, the overall quality of the thinking and the interaction in the small groups was not as high as expected.

C. THE JASPER PROJECT: ANCHORED INSTRUCTION OF MATHEMATICAL PROBLEM SOLVING

Anchored instruction has been developed by the Cognition and Technology Group at Vanderbilt (CTGV, 1997) in response to a problem observed in many students that reflects a lack of self-regulation, namely, the phenomenon of inert knowledge; that is, knowledge that is available in students' minds and can be recalled on request, but is not spontaneously accessed or applied in situations where it is relevant to solve new problems. The CTGV (1997; see also Van Haneghan et al., 1992) argued that classroom practices themselves are largely responsible for imparting inert knowledge to students. For instance, word problem solving in school typically consists of choosing the arithmetic operation to figure out the single correct answer to a mostly stereotyped and artificial problem statement. This contrasts sharply with what is going on in the real world, in which posing and defining problems—involving such cognitive regulatory activities as sense making, goal setting, planning, and decision making—are of major importance, whereas arithmetic operations serve mainly as tools to carry out a plan or to achieve a goal put forward by the problem solver. Anchored instruction has been designed as an attempt to support the acquisition of useful knowledge and skills by helping students "develop a sense of agency that allows them to identify and define problems and systematically explore solutions" (CTGV, 1997, p. 37). In other words, anchored instruction aims to foster students' self-regulation skills.

With this goal in mind, and applying videodisc technology, the starting point of anchored instruction has been the creation of rich, authentic, and interesting problem-solving contexts that can serve as the basis for the design of generative learning environments that offer students ample
opportunities for self-regulated activities such as problem posing, exploration, and discovery. Although the Vanderbilt group stressed that the application of anchored instruction does not necessarily require the use of technology, they nevertheless prefer to situate instruction in video-based anchors, because the medium allows a richer, more realistic, and more dynamic presentation of information than textual material. As such, a technology-based implementation of anchored instruction makes the learning environment more powerful (Bransford, Sherwood, Hasselbring, Kinzer, & Williams, 1990). Accordingly, the Vanderbilt group has developed videodisc-based complex problem spaces that enable learners to explore and model a problem space involving mathematical problems for extended periods of time and from a diversity of perspectives; the problem spaces offer opportunities for cooperative learning and discussion in small groups, as well as for individual and whole-class problem solving. However, these videodisc-based problem spaces are only one of the components that they believe to be important. Other crucial aspects include (1) the guidance provided by an expert teacher, who organizes and designs the learning experiences, who stimulates cooperative learning and small-group as well as whole-class discussion, and who explicitly addresses the culture of the classroom, and (2) the availability to children of rich and realistic sources of information (CTGV, 1997).

The series of 12 videodiscs for mathematics instruction in the upper primary school, called *The Adventures of Jasper Woodburry*, has already become rather well known (for a description, see CTGV, 1997). In the initial videodisc of this series, called "Journey to Cedar Creek," a person named Jasper Woodburry takes a river trip to see an old cabin cruiser he is considering purchasing. Jasper and the cruiser's owner test-run the cruiser, after which Jasper decides to purchase the boat. Because the boat's running lights are inoperative, Jasper must determine if he can get the boat to his home dock before sunset. Two major questions that form the basis of Jasper's decision are presented at the end of the disc: (1) Does Jasper have enough time to return home before sunset? and (2) Is there enough fuel in the boat's gas tank for the return trip? It is obvious that these types of problems are much more authentic and realistic than the typical traditional classroom tasks, and those used in most previous investigations like the one by Lester et al. (1989) discussed above.

Underlying the Jasper series are the following seven interrelated design principles, derived from previous research and chosen because they facilitate the elicitation of specific kinds of problem-solving activities in pupils (CTGV, 1997; Van Haneghan et al., 1992).

1. Video-based presentation format: In addition to allowing a richer and more realistic presentation of information than text, the video-based format also has some advantages over real-life contexts, simply
because the latter are not always practical, efficient, and well structured, and they are often difficult to organize in a school situation.

2. Narrative format with realistic problems: The presentation of the problem in the form of a story helps pupils create a meaningful context.

3. Generative structure: By having the students themselves generate the resolution of the story, their active involvement in the learning process is stimulated.

4. Embedded data design: By having all the information needed to solve the problem embedded in the story, students are enabled to take part in problem identification, problem formation, and pattern recognition activities that traditional word problem solving does not allow.

5. Problem complexity: The problems are intentionally made complex—sometimes involving up to 15 steps—so that students learn to deal with complexity that is typical of real problems, but is mostly lacking in traditional curricula.

6. Pairs of related adventures: Triplets of related adventures provide extra practice, afford discussions about transfer, and promote analogical reasoning.

7. Links across the curriculum: Embedding information from other domains helps extend mathematical thinking to those areas and encourages the integration of knowledge.

It is obvious that these design principles contribute to creating realistic and motivating learning sites that have the potential to evoke self-regulatory activities in pupils. For instance, the generative format of the Jasper adventures means that the stories end and that the student themselves must generate the problems to be solved, which solicits strategic behavior to assess and understand a story, and planning behavior and choice of actions, respectively, called orientation and organization in the cognitive self-regulation strategy of Lester et al. (1989) (see Table 2 herein). Similar self-generated thoughts and actions are elicited by the fact that the stories and the problems are complex, that all the data needed are embedded, and that related adventures are provided.

Initial studies with the Jasper series have produced encouraging results (CTGV, 1993; Van Haneghan et al., 1992). A baseline study revealed that sixth graders who were high-achievers in mathematics were very poor in their approach to complex application problems of the kind used in the Jasper series without instruction and mediation. According to the investigators this was not too surprising, because pupils rarely have the opportunity to engage in such complex problem solving. However, a subsequent controlled teaching experiment showed that videodisc-based anchored instruction can substantially improve pupils' problem-solving processes and
skills. Participants belonged to a fifth-grade class of above-average students. The first day of the experiment, the Jasper video was shown to all pupils and then they were pretested. After pretesting, pupils were assigned either to an experimental or a control group. Both groups received three additional teaching sessions. During these sessions the experimental group engaged in problem analysis, problem detection, and solution planning with respect to Jasper's trip-planning decisions, thereby intensively relying on videodisc/computer technology controlled by the instructor. In the control group, the usual teaching methods were applied to instruct students in solving traditional one- and two-step word problems that involved the same concepts as the Jasper adventure. Following instruction, pupils received two posttests—one consisting of traditional word problems and one about organizing information in the Jasper video for problem solving—and a transfer test that assessed pupils' abilities to identify, define, and solve problems presented on video similar to those posed in the Jasper series. This individually administered transfer test especially constitutes an indicator of children's cognitive self-regulation skills. The results of the study can be summarized as follows. On the traditional problems of the first posttest, the experimental pupils performed as well as the control students. In contrast to the control group, the experimental group showed significant gains from pretest to the second posttest. Most important in this context, however, are the results on the transfer test. The analysis of thinking aloud interview protocols relating to children's problem solving on video near-transfer problems showed significant transfer in the experimental group, but not in the control groups. Whereas the control children were generally unsuccessful in formulating and solving the transfer problems, the experimental pupils demonstrated great strength in identifying, stating, and solving the distance-rate-time and fuel problems embedded in the video.

Additional studies over the past 5 years have confirmed the initial positive results of anchored instruction based on the Jasper series and extended them to less skilled pupils (CTGV, 1997). For instance, a series of investigations that compared the learning and transfer outcomes of students who worked with the Jasper series with others who were taught the same concepts in the context of traditional one- and two-step word problems showed that Jasper pupils acquired a much higher ability to transfer their knowledge and skills to new, unfamiliar, and complex problems. In other words, Jasper-based learning fosters children's skills in self-regulating their problem-solving activities. Meanwhile, the results of a dissemination project in which Jasper-based anchored instruction was implemented in schools in Tennessee and nine surrounding states provided evidence that pupils not only perform better on measures of complex problem solving and attitudes to mathematics, but also on traditional standardized mathematics achievement tests. Finally, in more recent work,
the Cognition and Technology Group at Vanderbilt has redesigned the Jasper series by supplementing the different adventures with analog and extension problems. It has been demonstrated already that working with these analog and extension tasks has a substantial positive impact on pupils' understanding and on their abilities to self-regulate their solution processes with respect to new, unfamiliar problem situations (Schwartz, Goldman, Vye, Barron, & the CTGV, in press).

Although the terminology that is currently in vogue in the mainstream of research on self-regulated learning is not so explicitly used in the work and writings of the Cognition and Technology Group at Vanderbilt, it is nevertheless obvious that their Jasper Project is a very advanced attempt to fundamentally change mathematical instructional practices in the direction of fostering students' self-regulation of their learning, thinking, and problem-solving processes.

D. A POWERFUL LEARNING ENVIRONMENT FOR SKILLED REALISTIC MATHEMATICAL PROBLEM SOLVING IN THE UPPER ELEMENTARY SCHOOL

Taking into account the results and conclusions of the previous design studies, as well as the findings of a teaching experiment in their own center (Verschaffel & De Corte, 1997), Verschaffel (1999) developed a learning environment for skilled realistic mathematical problem solving in upper elementary-school children. Contrary to those earlier investigations, the Verschaffel et al. (1999) learning environment was realized and tested in a typical classroom context, rather than in a somewhat exceptional instructional setting wherein the teaching was done by a researcher and/or wherein advanced computer and video technology was brought into the school. As such, this study has a still higher degree of ecological validity than the preceding ones and, therefore, documents the practical relevance and utility of the underlying research-based ideas about the central role of heuristics and cognitive self-regulation in mathematical learning and problem solving.

The aims of the Verschaffel et al. (1999) learning environment are twofold. The first aim is the acquisition of an overall cognitive self-regulatory strategy for solving mathematics application problems consisting of five stages and involving a set of eight heuristic strategies that are especially useful in the first two stages of that strategy (see Table 4). It is obvious that the five stages of this strategy for cognitive self-regulation parallel the models proposed by Schoenfeld (1985) and by Lester et al. (1989) already presented.

The second aim is the acquisition of a set of appropriate beliefs and positive attitudes with regard to mathematical problem solving, as well as
TABLE 4  The Competent Problem-Solving Model Underlying the Learning Environment

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Build a mental representation of the problem</td>
</tr>
<tr>
<td></td>
<td>Heuristics: Draw a picture</td>
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<tr>
<td></td>
<td>Make a list, a scheme, or a table</td>
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<tr>
<td></td>
<td>Distinguish relevant from irrelevant data</td>
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<tr>
<td></td>
<td>Use your real-world knowledge</td>
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<tr>
<td>2</td>
<td>Decide how to solve the problem</td>
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<tr>
<td></td>
<td>Heuristics: Make a flowchart</td>
</tr>
<tr>
<td></td>
<td>Guess and check</td>
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<tr>
<td></td>
<td>Look for a pattern</td>
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<tr>
<td></td>
<td>Simplify the numbers</td>
</tr>
<tr>
<td>3</td>
<td>Execute the necessary calculations</td>
</tr>
<tr>
<td>4</td>
<td>Interpret the outcome and formulate an answer</td>
</tr>
<tr>
<td>5</td>
<td>Evaluate the solution</td>
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</table>

proper beliefs about the processes of teaching and learning it (e.g., “Mathematics problems may have more than one correct answer” or “Solving a mathematics problem may be effortful and take more than just a few minutes”).

The main features of the learning environment are the following:

1. A varied set of carefully designed complex, realistic, and challenging word problems that ask for the application of the intended heuristics and self-regulatory skills that constitute the model of skilled problem solving. As an illustration of the kind of problems used in the learning environments, two examples are given in the Figures 4 and 5.

2. A series of lesson plans based on a variety of teacher and learner activities. Each new component of the metacognitive strategy is initially modeled by the teacher. Furthermore, a lesson consists of a sequence of small-group problem-solving activities or individual assignments, always followed by a whole-class discussion. During all these activities the teacher’s role is to encourage and scaffold pupils to engage in and to reflect upon the kinds of cognitive and metacognitive activities involved in the model of competent mathematical problem solving. These encouragements and scaffolds are gradually withdrawn as the pupils become more competent and take more responsibility for their own learning and problem solving. In other words, external regulation is phased out as pupils become more self-regulated learners and problem solvers.
FIGURE 4  Example of a word problem used in the lesson about the heuristics “Use your real-world knowledge” (Step 1): Wim would like to attach a swing to a branch of a big old tree. The branch has a height of 5 meters. Wim already has made a suitable wooden shelf for his swing; Now Wim is going to buy some rope. How many meters of rope will Wim have to buy?

FIGURE 5  Example of a problem used in one of the project lessons: Pete and Annie are building a miniature town with cardboard. The space between the church and the town hall seems to be the perfect location for a big parking lot. The available space has the format of a square with a side of 50 cm and is surrounded by walls except for its street side. Pete already has made a cardboard square of the appropriate size. What will be the maximum capacity of their parking lot? 1. Fill in the maximum capacity of the parking lot on the banner. 2. Draw on the cardboard square how you can best divide the parking lot into parking spaces. 3. Explain how you came to your plan for the parking lot.
3. Interventions explicitly aimed at the establishment of new sociomathematical norms, resulting in a classroom climate that is conducive to the development in pupils of appropriate beliefs about mathematics and mathematics learning and teaching, and, by extension, to pupils' self-regulation of their learning. These norms relate to the role of the teacher and the pupils in the classroom (e.g., not the teacher on his or her own, but the whole class will decide which of the different learner-generated solutions is optimal after an evaluation of the pros and cons of the distinct alternatives), and about what counts as a good mathematical problem, a good solution procedure, or a good response (e.g., sometimes a rough estimate is a better answer than an exact number).

The learning environment consists of a series of 20 lessons designed by the research team in consultation and cooperation with the regular classroom teachers. Because the lessons were taught by the classroom teachers, they were prepared for and supported by implementing the powerful teaching–learning environment. With two lesson periods each week, the intervention was spread over about 3 months. Three major parts can be distinguished in the series of lessons:

1. Introduction to the content and organization of the learning environment and reflection upon the difference between a routine task and a real problem (1 lesson);
2. Systematic acquisition of the five-step regulatory problem-solving strategy and the embedded heuristics (15 lessons);
3. Learning to use the competent problem-solving model in a spontaneous and flexible way in so-called project lessons that involve more complex application problems (4 lessons).

The effectiveness of the learning environment to enhance active and self-regulated problem-solving in pupils was evaluated in a study with a pretest–posttest–retention test design. Four experimental fifth-grade classes and 7 comparable control classes from 11 different elementary schools in Flanders participated in the study. Three pretests were collectively administered in the experimental as well as the control classes: (1) a standardized achievement test (SAT) to assess fifth graders' general mathematical knowledge and skills, (2) a pretest consisting of 10 nonroutine word problems (WPT), and (3) a questionnaire aimed at assessing pupils' beliefs about and attitudes toward (teaching and learning) mathematical word problem solving (BAQ). Also, pupils' WPT answer sheets for each problem were analyzed carefully, looking for evidence of the application of one or more of the heuristics embedded in the problem-solving strategy. Also these collective pretests, three pairs of pupils of equal ability from each experimental class were asked to solve five nonroutine application
problems during a structured interview. The problem-solving processes of these dyads were video-registered, and afterward analyzed by means of a self-made schema for assessing the intensity and the quality of pupils' cognitive self-regulation activities. While the intervention took place in the experimental classes, the control classes followed the regular math program. By the end of the intervention, parallel versions of all collective pretests (SAT, WPT, and BAQ) were administered in all experimental and control classes. The answer sheets of all pupils were scrutinized again for traces of the application of heuristics, and the same pairs of pupils from the experimental classes as prior to the intervention were subjected again to a structured interview that involved parallel versions of the five nonroutine application problems used during the pretest. Major expectations were that as a result of acquiring the self-regulatory problem-solving strategy, the experimental pupils would significantly outperform the control children on the WPT, and that this would be accompanied by a significant increase in the use of heuristics; furthermore it was anticipated that the frequency and the quality of the self-regulation activities in the dyads would substantially grow. Three months later a retention test—a parallel version of the collective WPT used as pretest and posttest—was also administered in all experimental and control classes. To assess the implementation of the learning environment by the teachers of the experimental classes, a sample of four representative lessons was videotaped in each experimental class and analyzed afterward to get a so-called implementation profile for each experimental teacher.

The results of this study can be summarized as follows. First, although no significant difference was found between the experimental and control groups on the WPT during the pretest, the former significantly outperformed the latter during the posttest, and this difference in favor of the experimental group continued to exist on the retention test. However, it should be acknowledged that in the experimental group, pupils' overall performance on the posttest and retention tests was not as high as anticipated (i.e., the pupils of the experimental classes still produced only about 50% correct answers on these tests). Second, in the experimental group there was a significant improvement in pupils' beliefs about and attitudes toward (learning and teaching) mathematical problem solving, while in the control group there was no change in pupils' reactions to the BAQ from pretest to posttest. Third, although there was no difference between the pretest results on the SAT between the experimental and the control group, the results on the posttest revealed a significant difference in favor of the former group, indicating some transfer effect of the intervention toward mathematics as a whole. Fourth, a qualitative analysis of the pupils' response sheets for the WPT revealed a dramatic increase from pretest to posttest and retention test in the manifest use of (some of) the heuristics that were specifically addressed and discussed in the learning
environment; in the control classes there was no difference in pupils’ use of heuristics between the three testing moments. In line with this result, the videotapes of the problem-solving processes of the dyads revealed substantial improvement in the intensity and the quality with which the pairs from the experimental classes applied certain—but not all—(meta)cognitive skills that were specifically addressed in the learning environment. Both findings are indicative of a substantial increase in pupils’ ability to self-regulate their problem-solving processes. Fifth, although there is some evidence that pupils of high and medium ability benefited more from the intervention than low-ability pupils, the statistical analysis revealed at the same time that all three ability groups contributed significantly to all the abovementioned positive effects in the experimental group. This is a very important outcome, because it suggests that through appropriate intervention, the cognitive self-regulatory skills of the weaker children can be improved also. Finally, the positive effects of the learning environment were not observed to the same extent in all four experimental classes. Actually, in one of the four classes there was little or no improvement on most of the process and product measures. Analysis of the videotapes of the lessons in these classes indicated substantial differences in the extent to which the four experimental teachers succeeded in implementing the major aspects of the learning environment. For three of the four experimental classes, there was a good fit between the teachers’ implementation profiles, on the one hand, and their pupils’ learning outcomes, on the other hand.

In this intervention study a set of carefully designed application problems, a varied series of highly interactive teaching methods, and an attempt to change the sociomathematical classroom norms were combined in an attempt to create a powerful learning environment that focuses on the development of a mindful and self-regulated cognitive approach toward mathematical modeling and problem solving. The findings indicate that this intervention can have significant positive effects on different aspects of pupils’ mathematical modeling ability, on their cognitive self-regulation of and performance in problem solving, and on their beliefs about (learning and teaching) mathematics. However, as already mentioned, the effects were not as strong as expected; therefore, the results warrant less optimistic conclusions with respect to the possibility of changing and improving educational practice easily and quickly.

E. LOOKING BACK TO THE FOUR DESIGN EXPERIMENTS

All four design experiments have reported positive outcomes—albeit in different degrees—in terms of students’ performance. It is now interesting to ask in retrospect explicitly to what degree there are similarities among
Concerning the first issue, it was mentioned at the outset of this section that the design experiments focus mainly on students’ cognitive self-regulation. In three out of the four interventions, acquiring cognitive self-regulation was elaborated in terms of learning an overall metacognitive strategy consisting of a four- to five-step systematic approach to problem solving. It already has been mentioned that these strategies of Schoenfeld (1985), Lester et al. (1989), and Verschaffel et al. (in press) are very similar. Looking back at the results and contrasting the strategies with what happens in traditional mathematics classrooms, it seems to us that—without underestimating the significance of the strategy as a whole—the crucial self-regulatory components in the three approaches are the first steps. Although different terms are used in the three intervention studies, the regulatory activities of problem solving in the first two stages come more or less to the same thing, namely, strategic behavior aimed at understanding the problem by constructing a good representation of what it is about, and, thereafter, designing or making a solution plan that involves the use of cognitive strategies such as heuristics. For instance, in the study by Verschaffel et al., we have seen that after the intervention the spontaneous, self-regulated use of the heuristics taught in the first and second stage of the overall metacognitive strategy increased significantly in the experimental pupils. In accordance with this observation, Lester and his colleagues reported that orientation, the first phase of their strategy, had the most important effect on pupils’ problem solving. The work of Schoenfeld has shown that in his learning environment students’ problem-solving approach becomes more expertlike; as shown in Section III.A of this chapter, the major differences between novices and experts relate precisely to analysis of the problem and to solution planning. The Jasper project does not use a cognitive regulation strategy similar to the three other investigations. However, in their learning environment, the Vanderbilt group stresses precisely the same two cognitive self-regulatory skills when they argue that pupils “should develop a sense of agency that allows them to identify and define problems and systematically explore solutions” (CTGV, 1997, p. 44). With respect to the first aspect—problem analysis and understanding—they even go a step further by accentuating the importance of problem generating, and they heavily stress “the importance of helping students learn to plan, a process that involves generating subgoals necessary to solve complex problems.” (CTGV, 1997, p. 65)

The instructional models that underlie the learning environment of the four projects also share major components. In fact, the three main characteristics of the learning environment developed by Verschaffel et al. are
also largely reflected in the other interventions:

- Except for Schoenfeld's project, the other three use more realistic and challenging tasks than the traditional textbook word problems (the fact that this feature is less prominent in Schoenfeld's study probably has to do with the fact that he focuses on the teaching of geometry at the college level).

- All four projects apply a comparable variety of teaching methods and learner activities, including modeling of strategic aspects of problem solving by the teacher, guided practice with coaching and feedback, problem solving in small groups, and whole-class discussion focusing on evaluation and reflection concerning alternative solutions as well as different solution strategies. It is important in view of fostering cognitive self-regulation that in all interventions, external regulation is progressively withdrawn as students' competence increases.

- Finally, and related to the previous characteristic, all interventions create a classroom climate that is conducive to the development in pupils of appropriate beliefs about mathematics and mathematics learning and problem solving, and to active, self-regulated learning and problem solving.

As mentioned in the introductory paragraph of this section, Schunk (1998) has carried out a series of intervention studies that are more experimental in character than the four projects reported. It is nevertheless interesting to compare his instructional model with the learning environments of those four design experiments. The basic features of Schunk's (1998) intervention are modeled demonstrations, guided practice, self-regulatory training (e.g., goal setting, strategy use), and independent/self-reflective practice. Although these aspects are very convergent with the design experiments, there are apparently a number of important differences. First, whereas in Schunk's interventions the organization of self-regulatory activities happened in the context of learning basic arithmetic skills (e.g., subtraction, division, fractions, ...), the major objective of the four abovementioned design experiments was to enhance the development of a set of cognitive and self-regulatory skills for complex problem solving. In the second place, Schunk's (1998) interventions were implemented in a context "similar to that in which much mathematics instruction takes place in American elementary schools" (p.144). This seems to imply that traditional mathematics tasks and problems were used in an also traditional classroom context and climate. With regard to the teaching methods used in Schunk's intervention program, no mention is made of small-group work, which is a major component in all four design experiments described. It seems to us that for future researchers who undertake intervention studies within the framework of a specific theoreti-
V. CONCLUSIONS AND FUTURE DIRECTIONS FOR RESEARCH

In this chapter we have elucidated the importance and centrality of self-regulation in the context of the prevailing new conception of school mathematics, namely, as a major objective of mathematics education, on the one hand, and as a crucial characteristic of effective mathematics learning, on the other. However, this importance contrasts sharply with the many observations that demonstrate that students of different ages show serious shortcomings in the various self-regulatory aspects of a mathematical disposition, more specifically, in their metacognitive and metavolitional skills. In addition, they are affected by naive and incorrect beliefs about mathematics, mathematics learning, and problem solving that are not conducive to the development of self-regulation skills. In the preceding section we presented evidence that these weaknesses should not necessarily occur and develop in students, and, if they do, that they probably can be remedied. Indeed, a series of design experiments, in which new, research-based learning environments were implemented in real classrooms, indicate that such interventions can have significant favorable effects on students' cognitive self-regulation skills as well as on their mathematics-related beliefs. A comparison of the instructional models that underlie the interventions in those experiments led us to the identification of a rather coherent set of principles for the design of powerful learning environments. These principles relate to the nature of the tasks and problems presented to the pupils, the kinds of powerful instructional activities embedded in the learning environment, and the sociomathematical norms that determine the classroom culture and climate.

Notwithstanding the latter positive conclusion, major questions for continued research remain. First of all, this chapter illustrates that some aspects of self-regulated mathematics learning have so far not, or at least not sufficiently, been addressed. This is especially the case for the metavolitional skills, but also for the beliefs about oneself as a mathematics learner, which have an important impact on the development of pupils' self-regulation. However, even those aspects that have been studied are in need of further inquiry aimed at a better understanding of the most crucial processes involved in regulating learning and problem solving effectively and at tracing the development of regulatory processes in children and young adults. For instance, by comparison, we were able to identify major self-regulatory components in the overall metacognitive strategies used in three design experiments. However, future research is needed to further
unravel the skills and processes involved in regulating problem understanding and solution planning. In this respect, Zimmerman (1994) has argued rightly that the explanatory power of the construct of self-regulation is still rather restricted, that many of the self-regulatory processes that have been proposed, such as cognitive self-monitoring, are subtle and covert, and that major constructs that relate to self-regulation are conceptually overlapping.

The fact that a number of intervention studies have been carried out with respect to mathematics meets a recommendation that was made in the literature (Schunk & Zimmerman, 1994). These investigations show convincingly that by immersing students in a new, powerful learning environment, it is possible to foster their cognitive self-regulation skills. We even were able to identify a set of common components in those environments that relate to the nature of the problems, the instructional activities, and the classroom culture. However, the holistic design and implementation of these complex learning environments do not allow us to trace the relative significance of the different components in accounting for the observed positive effects. Therefore, there is also a strong need for studies that are intent on unravelling how and under what specific instructional conditions students become self-regulated learners in a more precise way. What are the crucial elements in the learning environment that help and support students in learning to manage and monitor their own processes of knowledge building and skill acquisition? In other words, how can the transition from external to self-regulation most effectively be enhanced? Also relative to this issue is the important question concerning the interaction between domain-specific knowledge and competence, on the one hand, and self-regulation, on the other: Does (successful) regulation of mathematics learning and problem solving require a certain level of mathematical competence? (Alexander, 1995).

REFERENCES


PART III. INTERVENTIONS AND APPLICATIONS OF SELF-REGULATION


